Int. J. Solids Structures, 1974, 10, pp. 135–136. Pergamon Press. Printed in Gt. Britain.

SOLUTION TO GEOMETRIC PROGRAMMING PROBLEMS BY TRANSFORMATION TO CONVEX PROGRAMMING PROBLEMS

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Abstract—Structural optimization problems which can be transformed to geometric programming problems can be easily solved by a further simple transformation to convex programming problems. Non-zero degree of difficulty geometric programming problems can be solved as easily as those with zero degree of difficulty.

In a recent paper[1] published in *Solids and Structures*, Morris showed how certain problems in structural optimization can be transformed to geometric programming problems and solved using geometric programming techniques. When the problem has a degree of difficulty greater than zero, the solution becomes quite involved and lengthy. It is well known that by using an exponential transformation, geometric programming problems can be changed into convex programming problems[2–4]. These transformed problems have a single optimum which can be easily found using a convex programming algorithm[5]. It is significant that geometric programming problems with non-zero degree of difficulty are handled just as easily as those with zero degree of difficulty. Thus, the procedure which is shown below provides an efficient method for solving the large class of structural optimization problems that Morris showed could be formulated in terms of posynomials.

Both of the problems which appear in the paper by Morris were solved using a problem oriented language (UHOP) being developed at the University of Houston. This language allows the user to state the problem in a non-formated manner and requires little knowledge of computer programming. Less than two seconds of computing time were required to solve both problems.

The zero degree of difficulty geometric programming problem used as an example by Morris is expressed in terms of posynomials as follows:

Minimize $W = 0{\cdot}188 x_1 x_3$

Subject to

$$1 \geqq 1{\cdot}75 x_1 x_2^{-1} x_3^{-1}$$

$$1 \geqq 900 x_1^{-2} + x_2^2 x_1^{-2}$$

By introducing the transformation

$$x_i = e^{T_i} \qquad i = 1, 2, 3$$

the following convex programming problem is obtained:

Minimize $W = 0{\cdot}188e^{(T_1+T_3)}$

Subject to

$$1 \geqq 1{\cdot}75e^{(T_1-T_2-T_3)}$$

$$1 \geqq 900e^{-2T_1} + e^{(2T_2-2T_1)}.$$

A more accurate solution was obtained for the problem on the optimal design of a ship bulkhead. Morris solved the problem using the dual relationship of geometric programming.

The solution obtained by using the convex algorithm is $x_1 = 57{\cdot}69$, $x_2 = 104{\cdot}44$, $x_3 = 57{\cdot}69$, $x_4 = 1{\cdot}05$, $x_5 = 34{\cdot}12$, with a minimum weight of $1{\cdot}365$.

The solution obtained by Morris differed in that $x_2 = 105{\cdot}52$, which gave a minimum weight of $1{\cdot}35$. However, his solution violates the second restricting equation in his paper

$$1 \geqq g(2) = \frac{26{\cdot}4(8{\cdot}94x_2)^{4/3}x_4^{-1}}{(2{\cdot}4x_1 + x_3)[x_3^2 - (x_2 - x_1)^2]}$$

by 14 per cent. Our solution violates this constraint by 2 per cent but satisfies the constraints of the approximating posynomials. In this example a 2 per cent error was introduced by approximating the original problem by posynomials.

In conclusion, we have found that Morris' method of approximating a class of structural optimization problems by posynomials is quite useful. However once a problem has been reduced to posynomial form we have found that the procedure of transforming the problem into a convex programming problem is much more efficient than the geometric programming approach used by Morris.

REFERENCES

1. A. J. Morris, Structural optimization by geometric programming. *Int. J. Solids Struct.* **8**, 847 (1972).
2. R. J. Duffin, E. L. Peterson and C. Zener, *Geometric Programming—Theory and Application.* Wiley (1967).
3. D. J. Wilde and C. S. Beightler, *Foundations of Optimization.* Prentice-Hall (1967).
4. S. L. S. Jacoby, J. S. Kowalik and J. T. Pizzo, *Iterative Methods for Nonlinear Optimization Problems.* Prentice-Hall (1972).
5. H. O. Hartley and R. R. Hocking, Convex programming by tangential approximation. *Management Sci.* **9**, 400 (1963).

Абстракт — Задачи определения оптимальных характеристик конструкции, которые преобразовуются к задачи геометрического программирования, можно легко решить путем добавочного простого преобразования к задачам выпуклого программирования. Задачи геометрического программирования, для не-нулевого порядка трудностей, решаются так же без труда как задачи для нулевого порядка трудностей.